A Serial Joint Channel and Physical Network Decoding

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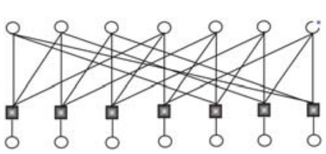
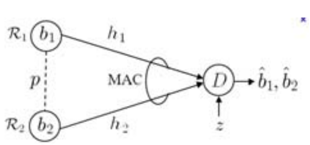




Outline

- A Serial Joint Channel and Physical Network Decoding
- PNC-aided STBC Scheme
- PNC-aided Low Density Lattice Code
- Conclusion

Physical Network Coding

- Interference has been traditionally viewed as harmful in wireless networks.
- Recently a technique called physical network coding [1], an important branch of network coding, has been proposed to enhance the throughput of networks by means of this wireless transmission characteristic.
  1. Amplify-and-forward
  2. Decode-and-forward [2]
  3. Compute-and-forward (Lattice code, Channel-coded PNC) [3][4]

[1] S. Zhang, S. C. Liew, P. P. Lam, “Hot Topic: Physical-layer Network Coding,” ACM MobiCom’06, pp. 358-365, Sept. 2006.

[2] P. Popovski, H. Yomo, "Bi-directional Amplification of Throughput in a Wireless Multi-Hop Network," Vehicular Technology Conference, 2006.

[3] B. Nazer, M. Gastpar, "Compute-and-Forward: Harnessing Interference Through Structured Codes," IEEE Trans on Information Theory, IEEE Transactions on , vol. 57, no.10, pp.6463-6486, Oct. 2011.

[4] S. Zhang and S. C. Liew, “Channel coding and decoding in a relay system operated with physical-layer network coding,” IEEE J. Sel. Areas Commun., vol. 27, no. 5, pp. 788–796, June 2009.

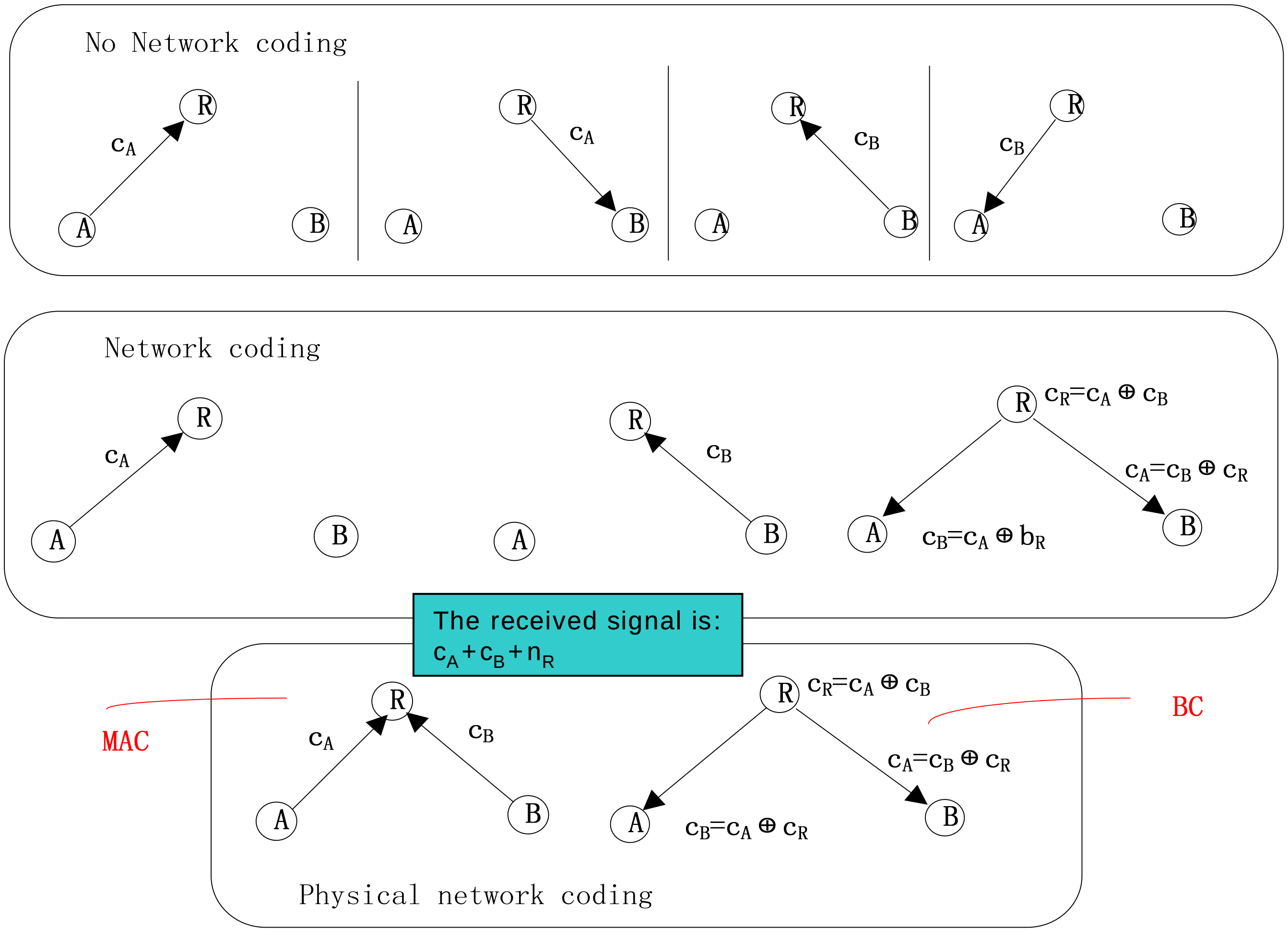
Developments

- BER analysis, outage probability, capacity analysis.
- Asynchronous physical-layer network coding
- MIMO/Cooperative PNC
- Channel-coded PNC

  Traditional channel coding based PNC

  Lattice code-based PNC

Physical Network Coding



Channel Coded PNC

- In the MAC stage, nodes A and B encoded b_A and b_B to c_A and c_B, modulated to x_A and x_B, which are transmitted simultaneously. The relay try to decode the received signal $y_R = x_A + x_B + n_R$ to $b_R = b_A \oplus b_B$,

- In the BC stage, b_R will be encoded and then broadcasted to A and B nodes。

- **We mainly focus on the performance in the MAC in that its performance is critical of the entire system [1][2].**

[1] S. Zhang and S.C. Liew, "Channel Coding and Decoding in a Relay System Operated with Physical-Layer Network Coding," *IEEE Trans on Selected Areas in Communications*, vol. 27, no. 5, pp. 788–796,Oct. 2009.

[2] Y. Lang and D.Wubben, "Generalized Joint Channel Coding and Physical Network Coding for Two-way Relay Systems," in *IEEE Proc. Vehicular Technology Conference* (VTC), Taipei, Taiwan, May 2010.

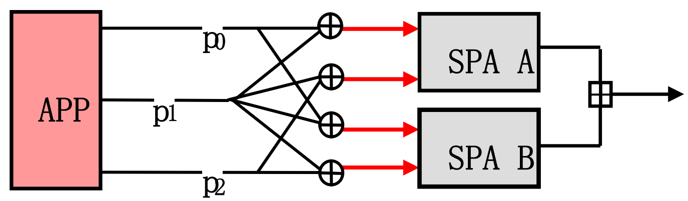
The decoding algorithm in the MAC

Table 1. The relationship between network-coded signal and the transmitted signal

$c_A(n)$	$c_B(n)$	$c_{A\oplus B}(n)$	$c_{A+B}(n)$	
0	0	0	0	p_0
0	1	1	1	p_1
1	0	1	1	
1	1	0	2	p_2

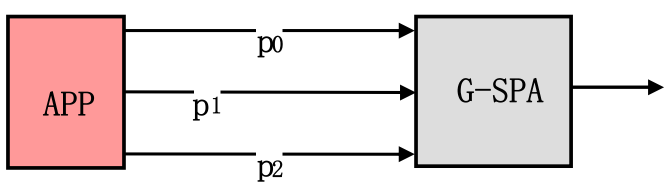
1. SCD (separated channel decoder, say MUD-XOR in [3])

This scheme is for the case that two nodes use the different channel codes.



2. G-JCNC (generalized joint channel decoding and physical network coding [1] [2])

Directly mapping $x_A+x_B+n_R \rightarrow c_R=c_A \oplus c_B$. Note that this scheme is for the case that two nodes use the same channel code.



[3] S. C. Liew, S. Zhang, and L. Lu, “Physical-layer network coding: tutorial, survey, and beyond,” *Physical Communication*.

The decoding algorithm in the MAC

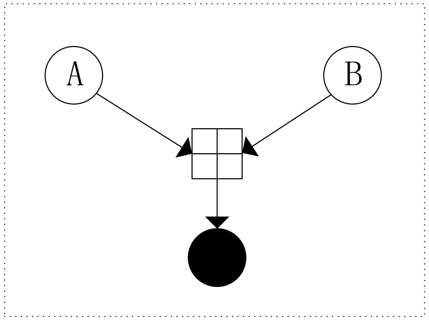
The drawback of the previous schemes:

- The previous G-JCNC scheme is only for the system using the same channel code, and put a constrain on transmission rate.
- For a system using the different channel codes, SCD is a alternative scheme but with the degraded performance.

- Thus, for the cases with the different channel codes, a serial joint channel and physical network decoding scheme (S-JCND) is proposed.

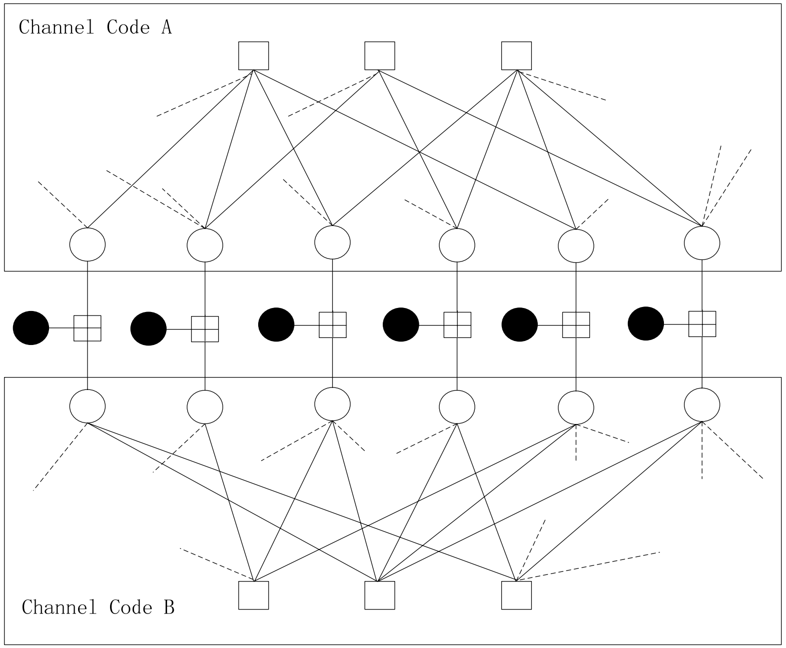
The S-JCND Scheme

The traditional SCD does not fully exploit the relationship $c_A + c_B = y$:



➢In the S-JCND, based on SCD, the messages out of the respective decoders are further updated according to this relationship.

➢The factor-graph of PNC system with the different source codes: Black nodes are the superimposed signals received at the relay.



The S-JCND Scheme

Table 1. The relationship between network-coded signal and the transmitted signal

$\mathbf{c}_{\mathrm{A}}(n)$	$\mathbf{c}_{\mathrm{B}}(n)$	$\mathbf{c}_{\mathrm{A}\oplus\mathrm{B}}(n)$	$\mathbf{c}_{\mathrm{A}+\mathrm{B}}(n)$	
0	0	0	0	$\mathbf{p}_0$
0	1	1	1	$\mathbf{p}_1$
1	0	1	1	$\mathbf{p}_1$
1	1	0	2	$\mathbf{p}_2$

The LLR of each bit in c_{A+B} being i, is denoted by p_i and computed by

$$\mathbf{p}_0(n) = \frac{1}{4\sqrt{2\pi}\Pr\{\mathbf{y}_{\mathrm{R}}(n)\}}\exp(-\frac{(\mathbf{y}_{\mathrm{R}}(n)-2)^2}{2\sigma_n^2}) \quad (2)$$

$$\mathbf{p}_1(n) = \frac{1}{2\sqrt{2\pi}\Pr\{\mathbf{y}_{\mathrm{R}}(n)\}}\exp(-\frac{\mathbf{y}_{\mathrm{R}}(n)^2}{2\sigma_n^2}) \quad (3)$$

$$\mathbf{p}_2(n) = \frac{1}{4\sqrt{2\pi}\Pr\{\mathbf{y}_{\mathrm{R}}(n)\}}\exp(-\frac{(\mathbf{y}_{\mathrm{R}}(n)+2)^2}{2\sigma_n^2}) \quad (4)$$

The initial LLR message for the two decoders:

$$\Pr\{\mathbf{c}_{\mathrm{M}}(n)=0|\mathbf{y}_{\mathrm{R}}(n)\} = \beta \times (\mathbf{p}_0(n) + 0.5 \times \mathbf{p}_1(n)) \quad (5)$$

$$\Pr\{\mathbf{c}_{\mathrm{M}}(n)=1|\mathbf{y}_{\mathrm{R}}(n)\} = \beta \times (\mathbf{p}_2(n) + 0.5 \times \mathbf{p}_1(n)) \quad (6)$$

The S-JCND Scheme

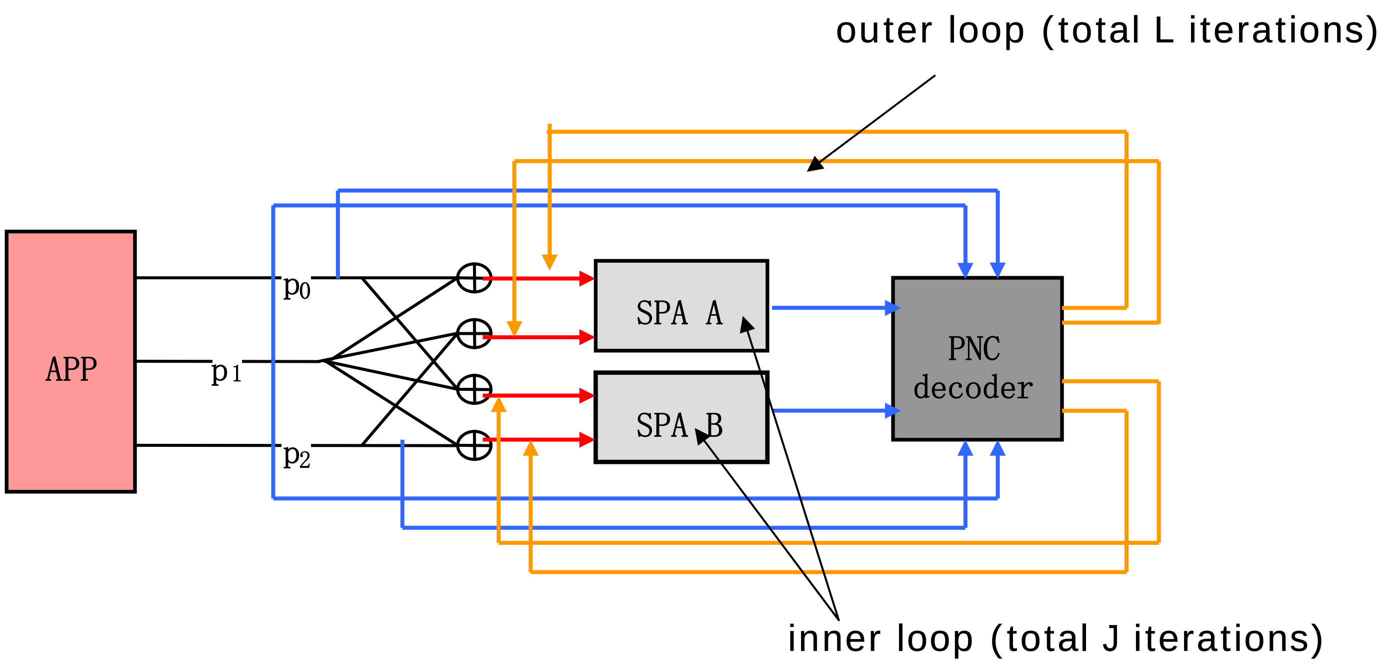


Fig 1. The Corresponding Decoding Diagram

The S-JCND Scheme

- Assuming the J and L denote the iteration number of outer and inner loop.

Step1: Initialization. Note the initial messages for A and B decoders are the same.

Step2: The decoders A and B are implemented by SPA algorithm using the input message. The resulting a-posterior messages, denoted by M_{A0}, M_{A1}, M_{B0} and M_{B1}, are fed into PNC decoder for further update.

Step3: The algorithm stops if the maximum outer iteration L is reached. Otherwise, the updated messages out of PNC decoder denoted by N_{A0}, N_{A1}, N_{B0} and N_{B1}, are fed into the A and B decoders for decoding, return to step 2.

- Decision. When iteration stops, the network-coded information can be obtained, i.e., $b_R = b'_A \oplus b'_B$. The b'_A and b'_B denote the estimated information vectors.

The S-JCND Scheme

Table 1. The relationship between network-coded signal and the transmitted signal

$\mathbf{c}_{\mathrm{A}}(n)$	$\mathbf{c}_{\mathrm{B}}(n)$	$\mathbf{c}_{\mathrm{A}\oplus\mathrm{B}}(n)$	$\mathbf{c}_{\mathrm{A}+\mathrm{B}}(n)$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	2

According the received signal between the two transmitted signals, the PNC update rule is as follows:

$$N_{A0}(n) = \beta_A \times (M_{B0}(n) \times p_0(n) + 0.5 \times M_{B1}(n) \times p_1(n))$$

$$N_{A1}(n) = \beta_A \times (M_{B1}(n) \times p_2(n) + 0.5 \times M_{B0}(n) \times p_1(n))$$

$$N_{B0}(n) = \beta_B \times (M_{A0}(n) \times p_0(n) + 0.5 \times M_{A1}(n) \times p_1(n))$$

$$N_{B1}(n) = \beta_B \times (M_{A1}(n) \times p_2(n) + 0.5 \times M_{A0}(n) \times p_1(n))$$

The S-JCND Scheme

Also note that for the A and B use the same linear channel code, an interleaver is used in either one of encoders to lower correlation between the two codebooks.

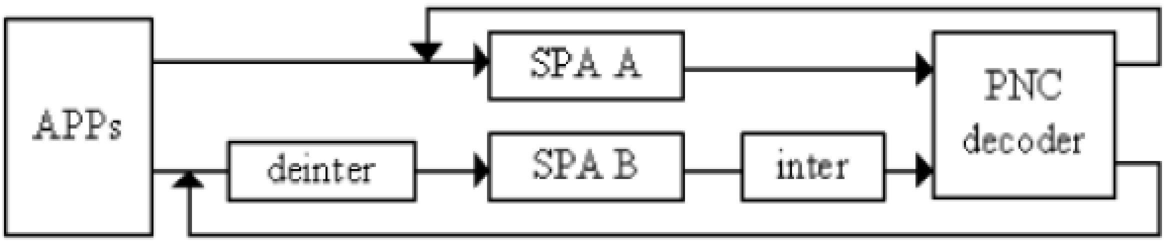


Fig. 2. The S-JCND in the cases with the same source channel codes.

Simulation Result

The two Repeat Accumulator codes (RA) are used. With the total iteration of 120, the BER performance are plotted, where the information length are 512 and 1024, codes rate are 0.25.

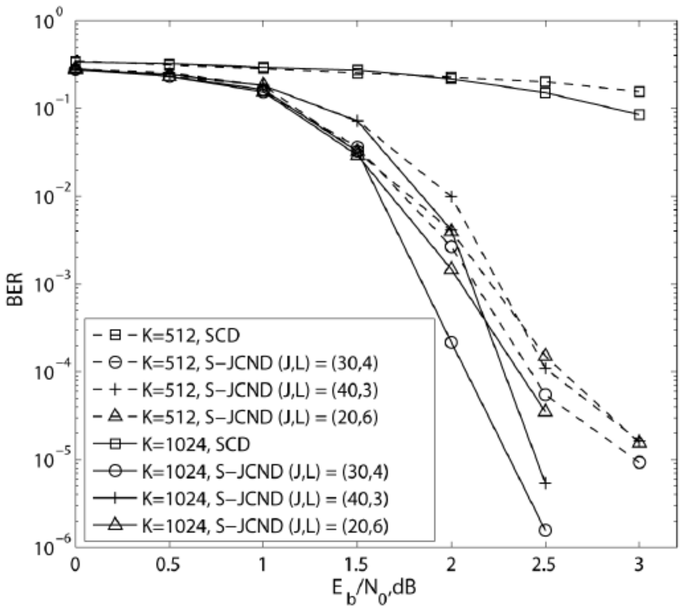


Fig. 3. The BER performance comparison between SCD and S-JCND with the different RA codes. Information lengths are 512 and 1024. Code rates are 0.25.

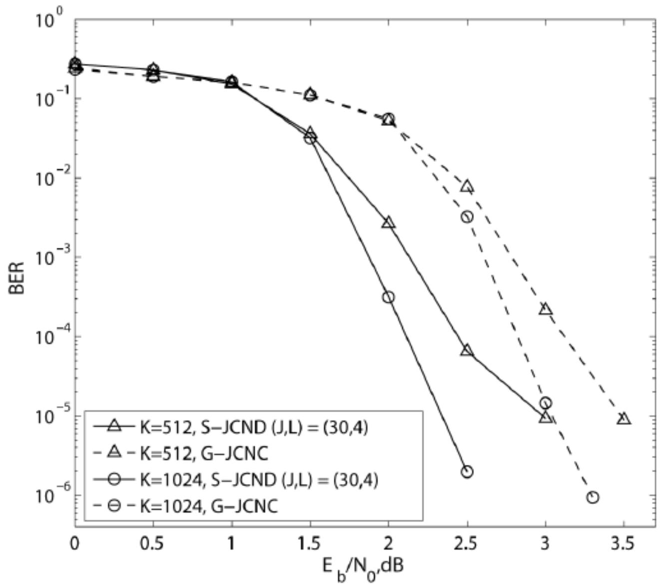


Fig. 4. The BER performance comparison between G-JCNC and S-JCND with the same RA codes. Information lengths are 512 and 1024. Code rates are 0.25.

- One can see that the proposed S-JCND outperforms the previous JCNC scheme about 0.5 dB.
- With the fixed iteration number, the values of (J, K) have an impact on the performance. The best performance is achieved when (J, L) = (30, 4).

Simulation analysis

➢The S-JCND with the different interleavers (s-rand, quasi-cyclic) are simulated.

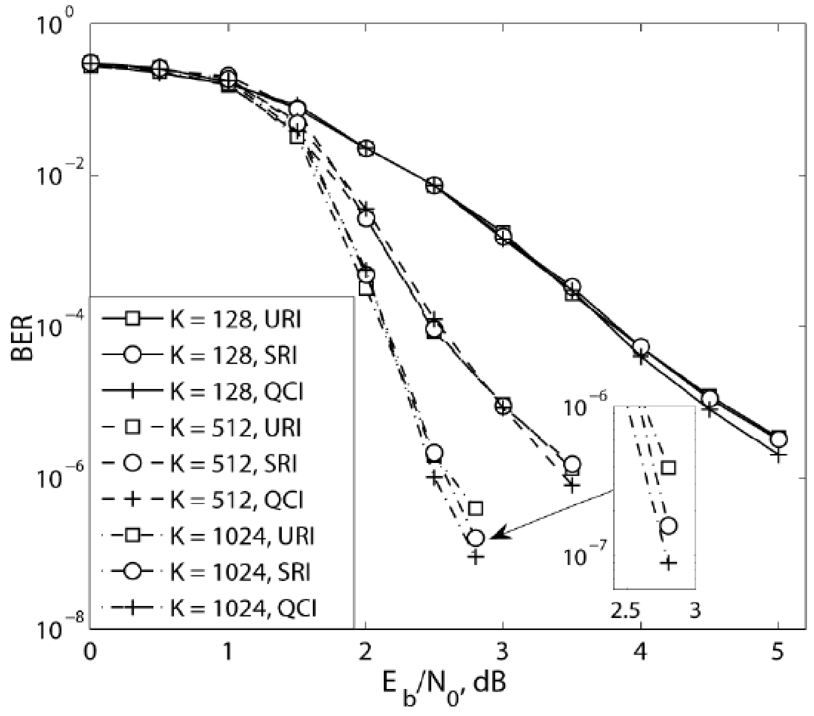


Fig. 5. The BER performance of S-JCND with different interleavers: URI, SRI and QCI, (J, L) = (30, 4). Information lengths are 128, 512 and 1024. Code rates are 0.25.

Conclusions

- It is similar to the joint network coding and channel decoding design in [1], where the scheme with the same codes has lower capacity than the one with the different codes.
- The results also prove a conclusion in [2], i.e., for low and medium code rates, we can recover both the combined as well as the individual messages.

- We will further demonstrate the results from the information-theoretic perspective.

[1] Shengli Zhang, Yu Zhu, Soung-Chang Liew and Letaief. Khaled Ben, "Joint Design of Network Coding and Channel Decoding for Wireless Networks," 2007, IEEE WCNC.
[2] S. Pfletschinger, "A practical physical-layer network coding scheme for the uplink of the two-way relay channel," Signals, Systems and Computers (ASILOMAR), the Forty Fifth Asilomar Conference, pp.1997-2001, 6-9 Nov. 2011

This work has been published: Pingping Chen and Lin Wang, “A serial joint channel and physical layer network decoding in two-way relay networks,” *IEEE communications letters*, vol. 16, no. 6, pp.769-772, 2012.

Outline

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- PNC-aided STBC Scheme
- PNC-aided Low Density Lattice Code
- Conclusion

STBC-PNC

System Model

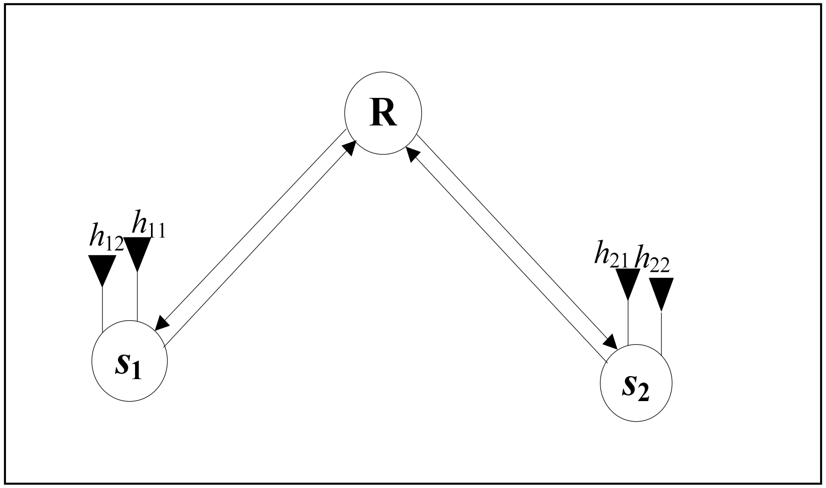


Fig. 1. PNC aided two-way relay network with two-antenna end nodes.

Each end node is equipped with Alamouti scheme of two transmit antennas. The relay has only one antenna.

[1] To. D, Choi. J and Kim. I, "Error probability analysis of bidirectional relay systems using Alamouti scheme," *IEEE Communications Letters,* vol. 14, no. 8, pp. 758-760, August 2010.

STBC-PNC

- Problem formulation

The transmitted blocks are
$$B_k = \begin{pmatrix} s_k(i-1) & -s_k^*(i) \\ s_k(i) & s_k^*(i-1) \end{pmatrix}$$

In the MA phase, the superimposed baseband signals are
$$\mathrm{y}^T = \sum_{k=1}^{2} \sqrt{E_k} H_k B_k + n_i, i = 1, 2,$$

and also written as
$$y_1 = s_1(i-1)h_{11} + s_1(i)h_{12} + s_2(i-1)h_{21} + s_2(i)h_{22} + n_1,$$
$$y_2 = -s_1^*(i)h_{11} + s_1^*(i-1)h_{12} - s_2^*(i)h_{21} + s_2^*(i-1)h_{22} + n_2.$$

- It is difficult to directly obtain the network-coded signals $s_{xor}(i\text{-}1) = s_1(i\text{-}1) \oplus s_2(i\text{-}1)$ and $s_{xor}(i) = s_1(i) \oplus s_2(i)$ from y_1 and y_2 using the traditional STBC combining decoding.
- The study[1] has proposed a scheme which calculated all possible combinations of the two transmitted signals。

STBC-PNC

$$\tilde{s}_{\mathrm{xor}}(l) = \underset{s_{\mathrm{xor}}(l)\in\mathcal{M}}{\mathrm{argmax}} \sum_{B_1,B_2:f(s_1(l),s_2(l))=s_{\mathrm{xor}}(l)} \exp(-\mathrm{T}(B_1,B_2)),$$

$$\mathrm{T}(B_1,B_2)) = \frac{||\mathrm{y}-H_1B_1-H_2B_2||^2}{\sigma^2}$$

This scheme results in considerably high computation complexity, especially when the high-order *M*-PSK modulation are adopted at the end nodes.

The relay computation complexity of the scheme [1] is $O(4^M)$.

To keep the relay relaxed, the STBC blocks are precoded before transmitted at the source nodes.

Goal: To directly obtain the network-coded signals rather than the individual signals in advance.

[1] To. D, Choi. J and Kim. I, "Error probability analysis of bidirectional relay systems using Alamouti scheme," *IEEE Communications Letters,* vol. 14, no. 8, pp. 758-760, August 2010.

Precoding STBC-PNC using global CSI

Transmitter

$$B_k = \begin{pmatrix} s_k(i-1) & -s_k^*(i) \\ s_k(i) & s_k^*(i-1) \end{pmatrix}, \qquad C_k = G_k B_k$$

B_K is STBC-coded block. G_k is precoding matrix, formed by CSI of the other channel.

$$G_1 = \begin{pmatrix} h_{21} & 0 \\ 0 & h_{22} \end{pmatrix} \text{ and } G_2 = \begin{pmatrix} h_{11} & 0 \\ 0 & h_{12} \end{pmatrix} \qquad \beta = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix},$$

The β is used to keep the transmitted power constant.

$$y_i = \sum_{k=1}^{2} \sqrt{E_k} H_k[\beta C_k] + n_i, i = 1, 2,$$

$$\beta_k^2 = \frac{E_b}{F_k \cdot F_k^*}, \quad k = 1, 2, \quad F_1 = (h_{11}\ h_{21}), F_2 = (h_{12}\ h_{22}),$$

Precoding STBC-PNC using global CSI

Receiver. The received signals:

$$
\begin{aligned}
y_1 &= \beta_1 h_{11} h_{21} s_1(i-1) + \beta_2 h_{12} h_{22} s_1(i) \\
&\quad + \beta_1 h_{11} h_{21} s_2(i-1) + \beta_2 h_{12} h_{22} s_2(i) + n_1 \\
&= h_1\{s_1(i-1) + s_2(i-1)\} + h_2\{s_1(i) + s_2(i)\} + n_1. \\
y_2 &= -\beta_1 h_{11} h_{21} s_1^*(i) + \beta_2 h_{12} h_{22} s_1^*(i-1) \\
&\quad - \beta_1 h_{11} h_{21} s_2^*(i) + \beta_2 h_{12} h_{22} s_2^*(i-1) + n_2 \\
&= -h_1\{s_1^*(i) + s_2^*(i)\} + h_2\{s_1^*(i-1) + s_2^*(i-1)\} + n_2.
\end{aligned}
$$

$$
\begin{aligned}
\tilde{s}_{\text{sum}}(i-1) &= h_1^* y_1 + h_2 y_2^* \\
&= (|h_1|^2 + |h_2|^2) s_{\text{sum}}(i-1) + h_1^* n_1 + h_2 n_2^* \\
&= t_{\text{sum}}(i-1) + h_1^* n_1 + h_2 n_2^*, \\
\tilde{s}_{\text{sum}}(i) &= h_2^* y_1 - h_1 y_2^* \\
&= (|h_1|^2 + |h_2|^2) s_{\text{sum}}(i) + h_1 n_2^* + h_2^* n_1, \\
&= t_{\text{sum}}(i) + h_1 n_2^* + h_2^* n_1,
\end{aligned}
$$

Precoding STBC-PNC using global CSI

- MAP Decoding (Max A-posterior Decoding)
  The computation complexity is $O(M^2)$, much lower than the complexity $O(4^M)$ in previous scheme .

$$\tilde{s}_{\text{xor}}(l) = \begin{cases} 1, & \text{if } \ln\left(\dfrac{\Pr\{s_{\text{xor}}(l) = 1|\tilde{s}_{\text{sum}}(l)\}}{\Pr\{s_{\text{xor}}(l) = 0|\tilde{s}_{\text{sum}}(l)\}}\right) \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

- Simplified Threshold Decision (TD) Decoding is derived. It has much reduced complexity by only using comparison operations.

BPSK:

$$\begin{aligned} \lambda &= \ln\left(\frac{\Pr\{s_{\text{xor}}(l) = 1|\tilde{s}_{\text{sum}}(l)\}}{\Pr\{s_{\text{xor}}(l) = 0|\tilde{s}_{\text{sum}}(l)\}}\right) \\ &= \ln\left(\exp\left(\frac{-4\text{C}}{\sigma_n^2}\right)\right) + \ln\left(\exp\left(\frac{4\tilde{s}_{\text{sum}}(l)}{\sigma_n^2}\right) + \exp\left(\frac{-4\tilde{s}_{\text{sum}}(l)}{\sigma_n^2}\right)\right) \\ &= \frac{-4\text{C}}{\sigma_n^2} + \ln\left(\exp\left(\frac{4|\tilde{s}_{\text{sum}}(l)|}{\sigma_n^2}\right)\right) + \ln\left(1 + \exp\left(\frac{-8|\tilde{s}_{\text{sum}}(l)|}{\sigma_n^2}\right)\right) \\ &= \frac{-4\text{C}}{\sigma_n^2} + \frac{4|\tilde{s}_{\text{sum}}(l)|}{\sigma_n^2} + \ln\left(1 + \exp\left(\frac{-8|\tilde{s}_{\text{sum}}(l)|}{\sigma_n^2}\right)\right) \qquad (25) \end{aligned}$$

with the small noise

$$\frac{-4\text{C}}{\sigma_n^2} + \frac{4|\tilde{s}_{\text{sum}}(l)|}{\sigma_n^2} \geq 0$$

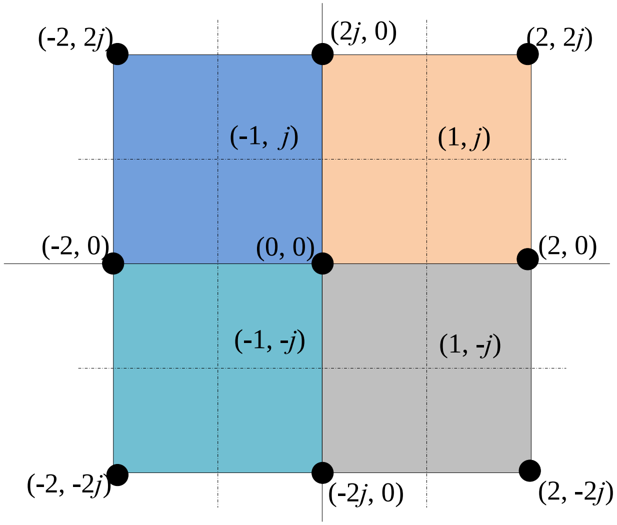
$$\Rightarrow |\tilde{s}_{\text{sum}}(l)| \geq \text{C}$$

$$\text{C} = (|h_1|^2 + |h_2|^2)$$

Achieve full two diversity orders

Precoding STBC-PNC using global CSI

QPSK:



TD Decoding for QPSK-based scheme can be similarly given by:

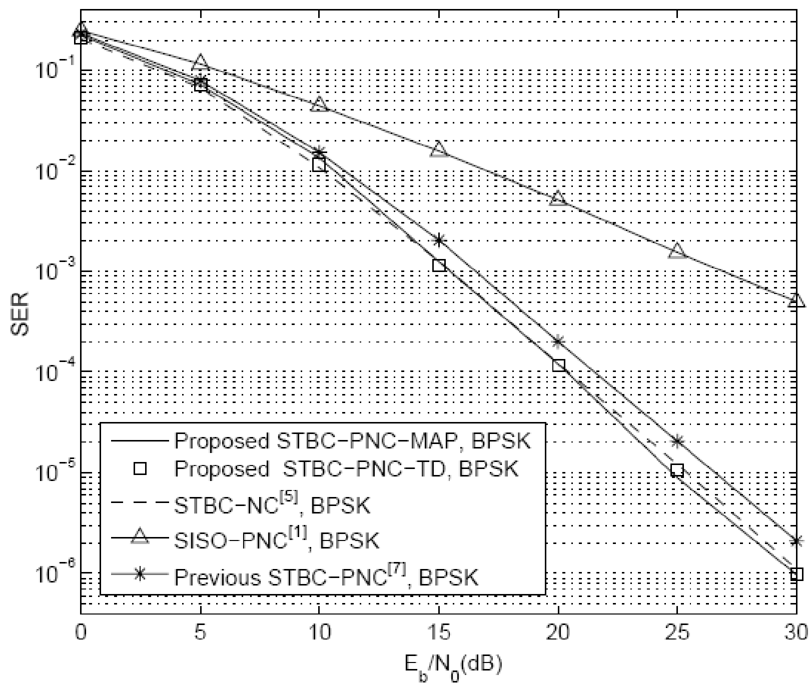
$$\tilde{s}_{\mathrm{xor}}(l) = \begin{cases} 00, & \text{if } (\lambda_1 \geq 1)\&\&(\lambda_2 \geq 1) \\ 01, & \text{if } (\lambda_1 \geq 1)\&\&(\lambda_2 < 1) \\ 10, & \text{if } (\lambda_1 < 1)\&\&(\lambda_2 \geq 1) \\ 11, & \text{if } (\lambda_1 < 1)\&\&(\lambda_2 < 1) \end{cases}$$

where

$$\lambda_1 = \mathrm{Re}\left(\frac{|\tilde{s}_{\mathrm{sum}}(l)|}{\mathrm{C}}\right)$$

$$\lambda_2 = \mathrm{Im}\left(\frac{|\tilde{s}_{\mathrm{sum}}(l)|}{\mathrm{C}}\right)$$

Simulation results



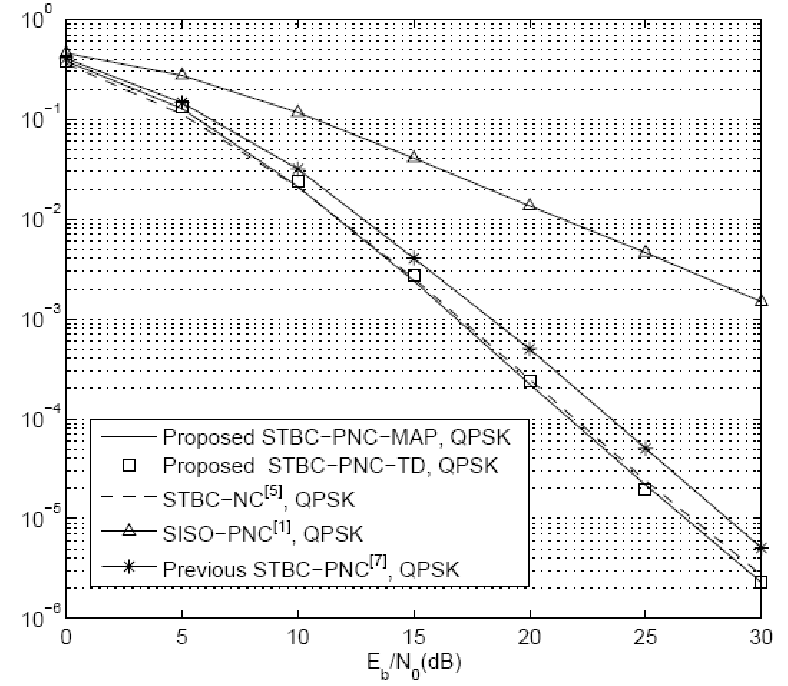


Fig. 2 SER performance of STBC-NC, SISO-PNC and the proposed STBC-PNC with two decoding algorithms: MAP and TD. The simulated systems are based on BPSK modulation.

Fig. 3. SER performance of STBC-NC, SISO-PNC and the proposed STBC-PNC with two decoding algorithms: MAP and TD. The simulated systems are based on QPSK modulation.

Outage probability of STBC-PNC

An achievable rate region of the STBC-PNC protocol is the closure of the convex hull of the set of points (R_1, R_2) satisfying the following inequalities:

$$R_1 < I_1^{PNC},$$
$$R_2 < I_2^{PNC},$$
$$R_1+R_2 < I_{sum}^{PNC}.$$

[1]S. J. Kim, P. Mitra, and V. Tarokh, “Performance bounds for bidirectional coded cooperation protocols,” *IEEE Trans. Inf. Theory*, vol. 54, pp. 5235–5241, Nov. 2008.
[2] Peng Liu and Il-Min Kim, "Outage probability analysis of physical-layer network coding in bidirectional relay networks," 25th *Biennial Symposium on QBSC*, pp.130-133, 12-14 May 2010.
[3] Peng Liu, Il-Min Kim, "Performance Analysis of Bidirectional Communication Protocols Based on Decode-and-Forward Relaying," *Communications, IEEE Transactions on* , vol.58, no.9, pp.2683-2696, September 2010

Outage probability of STBC-PNC

The outage probability of PNC-STBC system is

$$P_{\text{out}} = \Pr\left(I_1 < \frac{R}{2} \ \text{ or } I_2 < \frac{R}{2} \ \text{ or } I_{sum} < R \right)$$

where

$$I_1 = \min\left\{ \frac{1}{2}\log\left(1 + \frac{\theta\rho \| H_1 \|^2}{4} \right), \frac{1}{2}\log\left(1 + (1-\theta)\rho \| H_2 \|^2\right) \right\}$$

$$I_2 = \min\left\{ \frac{1}{2}\log\left(1 + \frac{\theta\rho \| H_2 \|^2}{4} \right), \frac{1}{2}\log\left(1 + (1-\theta)\rho \| H_1 \|^2\right) \right\}$$

$$I_{sum} = \frac{1}{2}\log\left(1 + \frac{\theta\rho(\| H_1 \|^2 + \| H_2 \|^2)}{4} \right)$$

where $\theta = Es/E$, $Es + Er = E$

The two-way complex channel variances are modeled as $\Omega_1 = d^{-4}$ and $\Omega_2 = (1 - d)^{-4}$.

➢A exact closed-form expression of the outage probability for the STBC-based PNC protocol is derived.

Outage probability of STBC-PNC system based on BPSK (R=1)

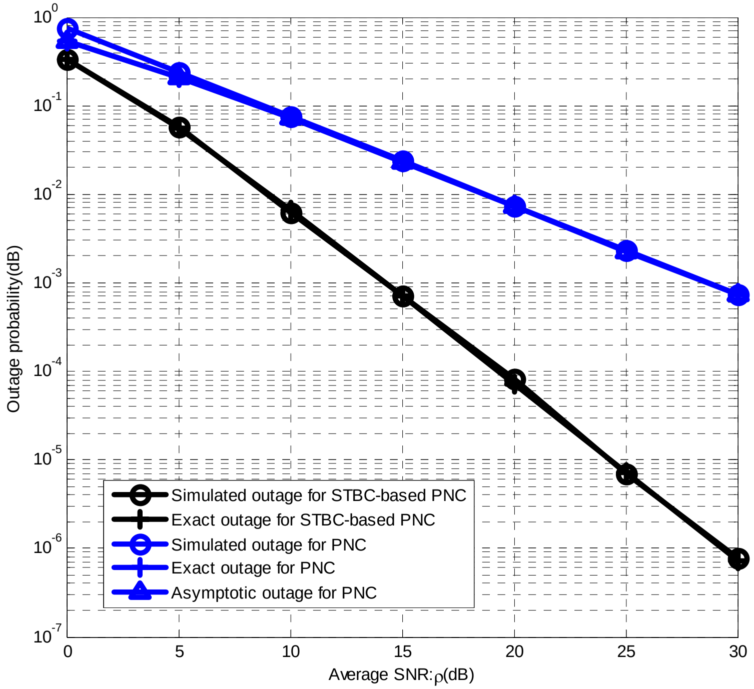


Fig.2. Outage probability against average SNR for d = 0.3, Es = 4E/5, and Er = E/5.

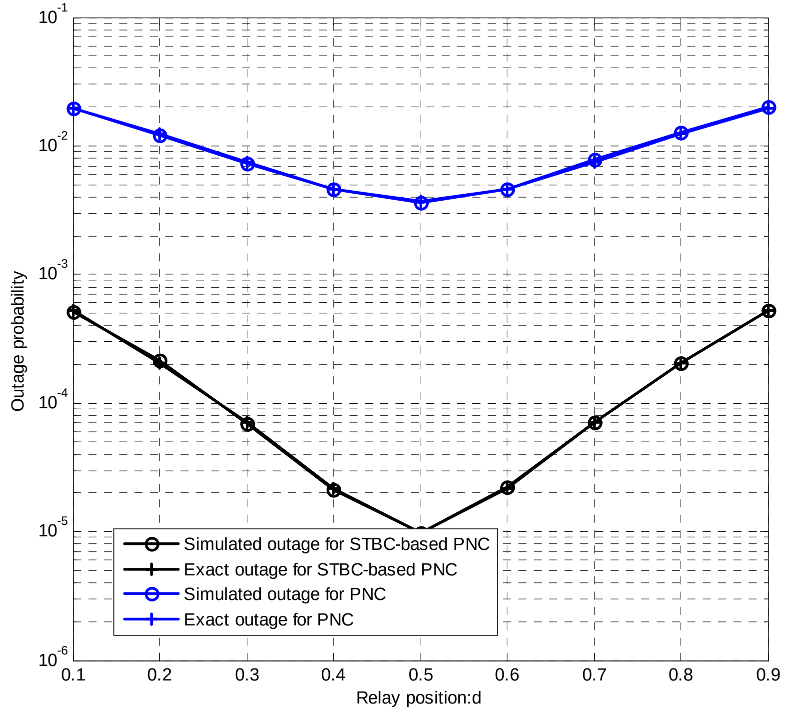


Fig.3. Outage probability for various relay position d while SNR = 20 dB, Es = 4E/5, and Er = E/5.

One can see that the simulation results are very close to the analytical ones.

Outline

- A Serial Joint Channel and Physical Network Decoding
- PNC-aided STBC Scheme
- PNC-aided Low Density Lattice Code (current work)
- Conclusion

PNC-aided Low Density Lattice Code (Current Work)

- The lattice coding have been studied for PNC scheme information-theoretically [1][2].

- It was shown that low density lattice code can also achieve capacity [3].

Goal: The design of joint low density lattice coding and physical network coding.

[1] M.P Wilson, K. Narayanan, H. D. Pfister, A. Sprintson, "Joint Physical Layer Coding and Network Coding for Bidirectional Relaying," Information Theory, IEEE Transactions on, vol.56, no.11, pp.5641-5654, Nov. 2010.
[2] B. Nazer, M. Gastpar, "Reliable Physical Layer Network Coding," Proceedings of the IEEE, vol. 99, no.3, pp.438-460, March 2011.
[3] N. Sommer, M. Feder, O. Shalvi, "Low-Density Lattice Codes," Information Theory, IEEE Transactions on, vol.54, no.4, pp.1561-1585, April 2004.

Low Density Lattice Code

- With power constrain *P* and noise variance σ^2, the AWGN channel capacity :

$$\tfrac{1}{2}\log(1+P/\sigma^2)$$

- Poltyrev defined the capacity of AWGN channel without restrictions; performance limited by *density* of the code-points.

For lattices, the density is determined by generator |*G*|. Poltyrev's capacity:

$$\sigma^2 < \frac{\sqrt[n]{|G|^2}}{2\cdot\pi\cdot e}$$

(with the det(|*G*|)=1 → the capacity becomes $\sigma^2 < 1/(2\pi e)$)

- With proper shaping and lattice decoding, a lattice achieving Poltyrev's capacity also obtains the AWGN capacity, at any SNR (Erez and Zamir)

[1] U. Erez, R. Zamir, "Achieving 1/2 log (1+SNR) on the AWGN channel with lattice encoding and decoding," Information Theory, IEEE Transactions on , vol.50, no.10, pp. 2293- 2314, Oct. 2004
[2] N. Sommer, M. Feder, O. Shalvi, "Low-Density Lattice Codes," Information Theory, IEEE Transactions on , vol.54, no.4, pp.1561-1585, April 2008

Low Density Lattice Code

Encoding:

- A lattice codeword is $\underline{\mathbf{x}} = G\underline{\mathbf{b}}$; b is a vector of integers. An n-dimensional lattice code is defined by its n*n lattice generator Matrix G.

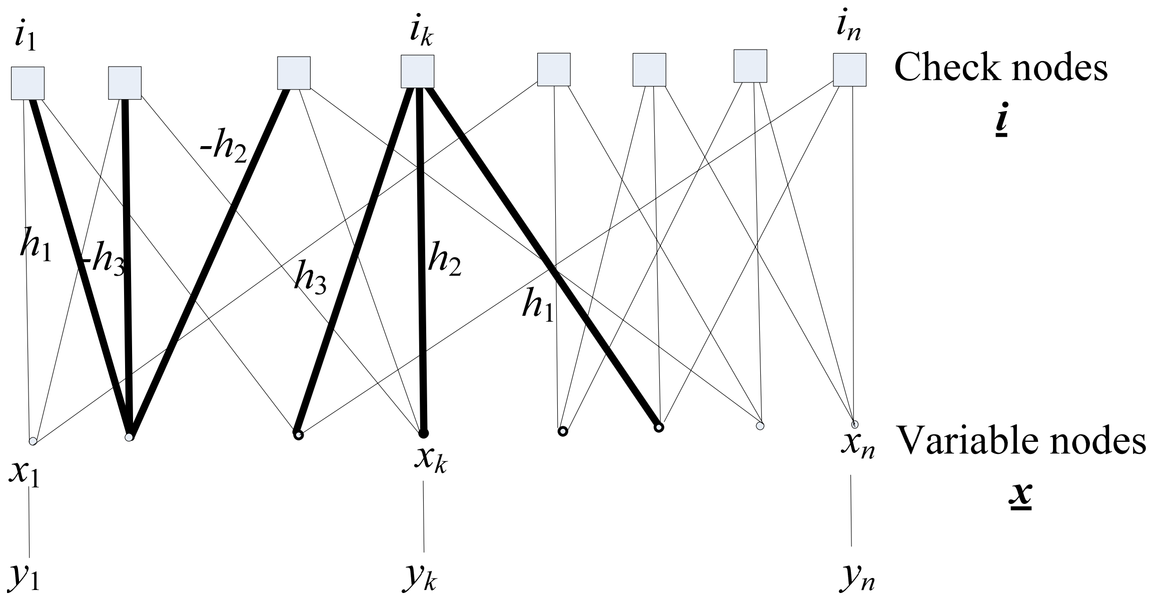
The parity matrix for lattice codes is defined as $H=G^{-1}$.

$$H\underline{\mathbf{x}} = G^{-1}\underline{\mathbf{x}} = \underline{\mathbf{b}} = \text{integer} \rightarrow \mathit{frac}\{H\underline{\mathbf{x}}\}=0$$

$\mathit{frac}\{x\}$ is the fractional part of x, defined as $\mathit{frac}\{x\} = x - [x]$.

Low Density Lattice Code

The bi-partite graph of LDLC



The received noisy vector:

$$\underline{y} = \underline{x} + \underline{n}$$

- A "*Latin Square LDLC" :* Regular LDLC where every row and column have the *same* non-zero values, except possible change in order and random signs

Iterative Decoding for LDLC

- An iterative scheme for calculating the PDF $f\,(x_k|\ \underline{\boldsymbol{y}}),\ k=1,\ldots,n$
- Message passing algorithm between variable nodes and check nodes.
- The check node : $\sum h_i x_{ki}$ = *integer .* (over real number field)
- The variable node : Get estimates of the considered variable PDF from the check nodes and the observation.

Advanced techniques for LDLC:

- Non-parametric belief propagation [1]
- Single-Gaussian messages estimation [2]
- LDLC codes constructed from protographs [3]

[1] D. Bickson, A. T. Ihler, H. Avissar, D. Dolev, "A low density lattice decoder via non-parametric belief propagation," *Communication, Control, and Computing, 2009. Allerton 2009. 47th Annual Allerton Conference on* , pp.439-446, Sept. 30 2009-Oct. 2 2009

[2] B. M. Kurkoski, K. Yamaguchi, K. Kobayashi, "Single-Gaussian messages and noise thresholds for decoding low-density lattice codes,“ Information Theory, ISIT 2009. IEEE International Symposium on, pp.734-738, June 28 2009.

[3] H. Uchikawa, B.M. Kurkoski, K. Kasai, K. Sakaniwa, "Threshold improvement of low-density lattice codes via spatial coupling," Computing, Networking and Communications (ICNC), International Conference on , pp.1036-1040, 2012.

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PNC-aided Low Density Lattice Codes

- For three-node PNC scheme, two nodes adopt the same G. The lattice codeword are $\mathbf{x}_1$ = Gb_1, $\mathbf{x}_2$ = Gb_2, $\mathbf{xx} = \mathbf{x}_1 + \mathbf{x}_2 = G(b_1+b_2)$ is a lattice codeword;

The received signals $\mathbf{y} = \mathbf{x}_1+\mathbf{x}_2+n$ can also be decoded.

Based on the non-parametric BP decoding, some theoretic and simulation results about the joint PNC-LDLC coding and decoding are under way.

Conclusions

- A S-JCND decoding with better performance.
- A precoding scheme for STBC-PNC using global CSI.
  Outage probability analysis for STBC-PNC scheme
- A LDLC coding based PNC scheme.

Further line:
I will focus on channel coding design, especially protograph-based coding, lattice coding for (MIMO) network coding and physical network coding.

Thanks for your attention!